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COMMENT

Parallel dynamics for an extremely diluted neural network

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Abstract. We revise our formula for the main overlap evolution at $t = 3$ generated by the parallel dynamics in the symmetric extremely diluted neural network. We show that for $t \geqslant 3$ correlations between the Gaussian and the Bernoulli's 'memory-like' noises have to be taken into account. To derive the exact formula for $t = 3$ we use our previously developed truncated dynamics method.

In our paper [1] we claimed that the recursion relation (27)* (hereafter * denotes formulae from [1]) gives an exact formula for the evolution of the main overlap for parallel dynamics in the extremely diluted (symmetric) neural network. The last step in our calculations (transition from (26)* to (27)*) was incorrect: the 'memory-like' term $\xi_i^{(q)} s_i(t = 1) g(t = 2)$ and Gaussian noise $\sqrt{\alpha}\mathcal{N}(0,1)$ are *correlated*. The simplest way to take this correlation into account is as follows.

Let us represent the Gaussian part of the noise in (26)* (or in (25)*) by using the 'truncated dynamics' [2]; then by definition (11)* and equations (21)*, (23)*, (25)*, we get

$$\text{`}\alpha\text{'-}\lim \xi_i^{(q)} s_i(t = 3) = \text{`}\alpha\text{'-}\lim \operatorname{sgn}\Bigg[m_N^{(q)}(t = 2) + \xi_i^{(q)} s_i(t = 1) g(t = 2) + \frac{1}{2c}\xi_i^{(q)} \sum_{p(\neq q)}^{M} \xi_i^{(p)} \sum_{j\in I_i} \xi_j^{(p)} \tilde{s}_j^{p}(t = 2)\Bigg] \tag{1}$$

where we use the 'truncated dynamics':

$$\tilde{s}_j^{p}(t = 2) = \operatorname{sgn}\Bigg[\xi_j^{(q)} m_N^{(q)}(t = 1) + \frac{1}{2c}\xi_j^{(p)} \sum_{k\in I_j\setminus i} \xi_k^{(p)} s_k(t = 1) + \frac{1}{2c} \sum_{f(\neq p,q)}^{M} \xi_j^{(f)} \sum_{k\in I_j} \xi_k^{(f)} s_k(t = 1)\Bigg]. \tag{2}$$

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Recall that for the 'memory-like' noise in (1) one has

$$\xi_i^{(q)} s_i(t=1) = \mathrm{sgn}\left[m_N^{(q)}(0) + \frac{1}{2c}\xi_i^{(q)} \sum_{p(\neq q)}^{M} \xi_i^{(p)} \sum_{j\in I_i} \xi_j^{(p)} s_j(t=0) \right]. \tag{3}$$

Then it becomes clear that it is correlated with the 'Gaussian-like' noise in (1) because variables $s_j(t=0)$ in (3) and $\widetilde{s}_j^p(t=2)$ in (1) are correlated. Remark that 'truncated dynamics' (2) leaves $\{\xi_i^{(p)}\xi_j^{(p)}\widetilde{s}_j^p(t=2)\}_{j,p}$ to be uncorrelated.

Now let us define two subsets of I_i relevant to dynamics (2): $I_{i\pm} = \{j \in I_i : \widetilde{s}_j^p(t=2)s_j(t=0) = \pm 1\}$. Then for the 'Gaussian-like' noise in (1) we get

$$\text{'}\alpha\text{'-lim}\left\{ \frac{1}{2c}\xi_i^{(q)} \sum_{p(\neq q)}^{M} \xi_i^{(p)} \Big(\sum_{j\in I_{i,+}} \xi_j^{(p)} s_j(t=0) - \sum_{j\in I_{i,-}} \xi_j^{(p)} s_j(t=0)\Big)\right\}$$
$$= \sqrt{\alpha p_+}\mathcal{N}_+(0,1) - \sqrt{\alpha p_-}\mathcal{N}_-(0,1) \tag{4}$$

where $p_\pm = \Pr\{\widetilde{s}_j^p(t=2)s_j(t=0) = \pm 1\}$ and $\mathcal{N}_\pm(0,1)$ are independent Gaussian noises with zero mean and variance 1. Similarly we obtain for the noise in (3) that

$$\text{'}\alpha\text{'-lim}\left\{ \frac{1}{2c}\xi_i^{(q)} \sum_{p(\neq q)}^{M} \xi_i^{(p)} \sum_{j\in I_i} \xi_j^{(p)} s_j(t=0)\right\} = \sqrt{\alpha p_+}\mathcal{N}_+(0,1) + \sqrt{\alpha p_-}\mathcal{N}_-(0,1). \tag{5}$$

Now, using representations (4), (5) and equations (1), (2), one gets

$$m^{(q)}(t=3) = \mathbb{E}(\xi_i^{(q)} s_i(t=3))$$
$$= \frac{1}{2\pi}\iint_{-\infty}^{+\infty} \mathrm{d}x\,\mathrm{d}y\, \exp[-(x^2+y^2)/2]\,\mathrm{sgn}\{m^{(q)}(t=2) + g(t=2)$$
$$\times\, \mathrm{sgn}[m^{(q)}(0) + \sqrt{\alpha p_+}x + \sqrt{\alpha p_-}y] + \sqrt{\alpha p_+}x - \sqrt{\alpha p_-}y\}. \tag{6}$$

Formula (6) can be rewritten in the form

$$m^{(q)}(t=3) = \frac{1}{2\sqrt{2\pi\alpha}} \sum_{\sigma=\pm1} \left[\int_{-\sigma g(t=2)-m^{(q)}(t=2)}^{+\infty} \mathrm{d}x - \int_{-\infty}^{-\sigma g(t=2)-m^{(q)}(t=2)} \mathrm{d}x \right]$$
$$\times \exp(-x^2/2\alpha)\left\{ 1 + \sigma\,\mathrm{erf}\left[\frac{(p_+ - p_-)x + m^{(q)}(t=0)}{2\sqrt{\alpha p_+ p_-}} \right]\right\}. \tag{7}$$

To calculate the probabilities $p_\pm$ one has to use the representation (2) and equation (17)*. Then

$$\widetilde{s}_j^p(t=2)s_j(t=0) = \text{'}\alpha\text{'-lim}\,\mathrm{sgn}\left\{ s_j(t=0)\xi_j^{(q)} m_N^{(q)}(t=1) + s_j(t=0)\frac{1}{2c}\xi_j^{(p)} \right.$$
$$\left. \times \sum_{k\in I_j\setminus i} \xi_k^{(p)} s_k(t=1) + s_j(t=0)\xi_j^{(q)} v_{j,I_j}^{p,q}(t=1) \right\} \tag{8}$$

where $v^{p,q}_{j,I_j}(t=1)$ is defined by (2). On the other hand by (17)* we have

$$\text{`}\alpha\text{'-lim}\, v^{p,q}_{j,I_j}(t=1) \stackrel{\mathrm{d}}{=} s_j(t=0)\xi_j^{(q)} g(t=1) + \sqrt{\alpha}\mathcal{N}(0,1) \tag{9}$$

where Gaussian and 'memory-like' noises are independent. Therefore, by (8) and (9) we obtain

$$\tilde{s}^p_j(t=2)s_j(t=0) = \mathrm{sgn}\{s_j(t=0)\xi_j^{(q)} m^{(q)}(t=1) + g(t=1) + \sqrt{\alpha}\mathcal{N}(0,1)\}. \tag{10}$$

Using (10) one gets

$$\begin{aligned} p_+ &= \Pr\{s_j(t=0)\xi_j^{(q)} m^{(q)}(t=1) + g(t=1) + \sqrt{\alpha}\mathcal{N}(0,1) > 0\} \\ &= \frac{1}{2}\left\{1 + \sum_{\sigma=\pm 1} \frac{1+\sigma m^{(q)}(0)}{2}\,\mathrm{erf}\left[\frac{\sigma m^{(q)}(t=1) + g(t=1)}{\sqrt{\alpha}}\right]\right\} \end{aligned} \tag{11}$$

and $p_- = 1 - p_+$.

Substituting (11) in (7) gives the formula that (up to simplifications) coincides with the one for $t = 3$ obtained recently for the same model by Watkin and Sherrington [3] using the Gardner–Derrida–Mottishaw method [4].

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